JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 18, No. 6, November–December 1995

Feedback Control Logic for Spacecraft Eigenaxis Rotations Under Slew Rate and Control Constraints

Bong Wie* and Jianbo Lu†
Arizona State University, Tempe, Arizona 85287

The problem of reorienting a rigid spacecraft as fast as possible within the physical limits of actuators and sensors is investigated. In particular, a nonlinear feedback control logic that accommodates the actuator and sensor saturation limits is introduced. The near-minimum-time eigenaxis reorientation problem of the X-Ray Timing Explorer spacecraft under slew rate and control torque constraints is used as an example to demonstrate the effectiveness and simplicity of the proposed nonlinear feedback control logic.

I. Introduction

SPACECRAFT are sometimes required to maneuver as fast as possible within the physical limits of actuators and sensors. The X-Ray Timing Explorer (XTE) spacecraft to be launched in 1996 is one of such spacecraft, and it will be required to maneuver about an inertially fixed axis as fast as possible, but within the saturation limit of rate gyros.[1] The XTE spacecraft will be controlled by a set of skewed reaction wheels; thus, the maximum available control torque also needs to be considered in such a near-minimum-time eigenaxis maneuver.

The eigenaxis rotation is in general not time optimal as discussed in Ref. 2. However, this paper is concerned with the practical problem of reorienting a rigid spacecraft about an inertially fixed axis as fast as possible, but within the saturation limits of reaction wheels and rate gyros.

Large-angle slew maneuver logic of many other spacecraft, including the recently flown Clementine spacecraft, is often driven by wheel momentum as well as torque saturation, but not by rate gyro saturation. However, the rest-to-rest slew control problem of the XTE spacecraft is mainly driven by the slew rate constraint caused by the saturation limit of low-rate gyros. Also, the angular momentum change of the XTE spacecraft during its planned slew maneuvers is well within its wheel momentum saturation limit. Consequently, the angular momentum of relatively small reaction wheels is ignored in the subsequent analysis in this paper. It is also noted that the proposed control logic is not unreasonably sensitive to inertia uncertainties since it has the same level of stability robustness as the various quaternion feedback control logic analyzed in Refs. 3 and 4.

The remainder of this paper is organized as follows. Section II briefly introduces the globally asymptotically stabilizing, quaternion feedback control logic. Section III treats the eigenaxis rotation of a rigid spacecraft without slew rate and control constraints. In Sec. IV, a normalized saturation function of an n-dimensional vector is defined and an m-layer cascade-saturation control algorithm is introduced. Section V is concerned with the problem of eigenaxis rotation under slew rate constraint, whereas Sec. VI treats the eigenaxis rotation problem with both slew rate and control torque constraints. Finally, in Sec. VII, the near-minimum-time eigenaxis reorientation problem of the XTE spacecraft under slew rate and control torque constraints is used as an example to demonstrate the effectiveness and simplicity of the proposed nonlinear feedback control logic.

II. Quaternion Feedback Control

Consider the attitude dynamics of a rigid spacecraft described by Euler's rotational equation of motion:

$$J\dot{\omega} + \omega \times J\omega = u \tag{1}$$

where J is the inertia matrix, $\omega = [\omega_1\ \omega_2\ \omega_3]^T$ the angular velocity vector, and $u = [u_1\ u_2\ u_3]^T$ the control torque input vector. The cross product of two vectors is represented in matrix notation as

$$\omega \times h \equiv \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

where $h = J\omega$ is the angular momentum vector. It is assumed that the angular velocity vector components ω_i along the body-fixed control axes are measured by rate gyros.

Euler's rotational theorem states that the rigid-body attitude can be changed from any given orientation to any other orientation by rotating the body about an axis, called the Euler axis, that is fixed to the rigid body and stationary in inertial space. Such a rigid-body rotation about an Euler axis is often called the eigenaxis rotation.

Let a unit vector along the Euler axis be denoted by $e = [e_1\ e_2\ e_3]^T$ where e_1, e_2, and e_3 are the direction cosines of the Euler axis relative to either an inertial reference frame or the body-fixed control axes. The four elements of quaternions are then defined as

$$q_1 = e_1 \sin(\theta/2) \tag{2a}$$

$$q_2 = e_2 \sin(\theta/2) \tag{2b}$$

$$q_3 = e_3 \sin(\theta/2) \tag{2c}$$

$$q_4 = \cos(\theta/2) \tag{2d}$$

where θ denotes the rotation angle about the Euler axis, and we have

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$$

The quaternion kinematic differential equations are given by

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{3}$$

Similar to the Euler-axis vector $e = [e_1\ e_2\ e_3]^T$, define a quaternion vector $q = [q_1\ q_2\ q_3]^T$ such that

$$q = e \sin(\theta/2)$$

Presented as Paper 94-3563 at the AIAA Guidance, Navigation, and Control Conference, Scottsdale, AZ, Aug. 1–3, 1994; received Sept. 21, 1994; revision received March 24, 1995; accepted for publication March 24, 1995.

*Professor, Department of Mechanical and Aerospace Engineering. Associate Fellow AIAA.

†Graduate Research Assistant, Department of Mechanical and Aerospace Engineering; currently Ph.D. Candidate, School of Aeronautics and Astronautics, Purdue University, Lafayette, IN 47907. Student Member AIAA.

Then Eq. (3) can be rewritten as

$$\dot{\boldsymbol{q}} = -\tfrac{1}{2}\boldsymbol{\omega} \times \boldsymbol{q} + \tfrac{1}{2}q_4\boldsymbol{\omega} \tag{4a}$$

$$\dot{q}_4 = -\tfrac{1}{2}\boldsymbol{\omega}^T\boldsymbol{q} \tag{4b}$$

where

$$\boldsymbol{\omega} \times \boldsymbol{q} \equiv \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Since quaternions are well suited for onboard real-time computation, spacecraft orientation is now commonly described in terms of the quaternions and a linear state feedback controller of the following form can be considered for real-time implementation:

$$\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{q} - \boldsymbol{C}\boldsymbol{\omega} \tag{5}$$

where $\boldsymbol{K}$ and $\boldsymbol{C}$ are controller gain matrices to be properly determined.

If the reference or commanded attitude quaternion vector is given as

$$\boldsymbol{q}_c = [q_{1c}\ \ q_{2c}\ \ q_{3c}]^T \neq [0\ \ 0\ \ 0]^T$$

then the control logic (5) can be simply modified to the following form:

$$\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{q}_e - \boldsymbol{C}\boldsymbol{\omega} \tag{6}$$

where $\boldsymbol{q}_e = [q_{1e}\ q_{2e}\ q_{3e}]^T$ is called the attitude error quaternion vector. The commanded attitude quaternions $(q_{1c}, q_{2c}, q_{3c}, q_{4c})$ and the current attitude quaternions (q_1, q_2, q_3, q_4) are related to the attitude error quaternions $(q_{1e}, q_{2e}, q_{3e}, q_{4e})$, as follows:

$$\begin{bmatrix} q_{1e} \\ q_{2e} \\ q_{3e} \\ q_{4e} \end{bmatrix} = \begin{bmatrix} q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c} \\ q_{1c} & q_{2c} & q_{3c} & q_{4c} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

It was shown in Refs. 3 and 4 that the closed-loop nonlinear system of a rigid spacecraft with the linear state feedback controller of the form (5) or (6) is globally asymptotically stable for the following gain selections:

Controller 1: $\boldsymbol{K} = k\boldsymbol{I}$; $\boldsymbol{C} = \mathrm{diag}(c_1, c_2, c_3)$
Controller 2: $\boldsymbol{K} = (k/q_4^3)\boldsymbol{I}$; $\boldsymbol{C} = \mathrm{diag}(c_1, c_2, c_3)$
Controller 3: $\boldsymbol{K} = k\,\mathrm{sgn}(q_4)\boldsymbol{I}$; $\boldsymbol{C} = \mathrm{diag}(c_1, c_2, c_3)$
Controller 4: $\boldsymbol{K} = [\alpha\boldsymbol{J} + \beta\boldsymbol{I}]^{-1}$; $\boldsymbol{K}^{-1}\boldsymbol{C} > 0$

where k and c_i are positive scalar constants, $\boldsymbol{I}$ is a 3×3 identity matrix, $\mathrm{sgn}(\cdot)$ denotes the signum function, and α and β are nonnegative scalars. Note that controller 1 is a special case of controller 4.

III. Eigenaxis Rotation

The gyroscopic term of Euler's rotational equation of motion is not significant for most practical rotational maneuvers. However, in some cases, it may be desirable to directly counteract the term by control torque, as follows:

$$\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{q} - \boldsymbol{C}\boldsymbol{\omega} + \boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega} \tag{7}$$

It was shown in Ref. 4 that the closed-loop system with the controller (7) is globally asymptotically stable if the matrix $\boldsymbol{K}^{-1}\boldsymbol{C}$ is positive definite. A natural selection of $\boldsymbol{K}$ and $\boldsymbol{C}$ for guaranteeing such condition is $\boldsymbol{K} = k\boldsymbol{J}$ and $\boldsymbol{C} = c\boldsymbol{J}$ where k and c are positive scalar constants to be properly selected. Furthermore, a rigid spacecraft with the controller

$$\boldsymbol{u} = -k\boldsymbol{J}\boldsymbol{q} - c\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega} \tag{8}$$

performs a rest-to-rest reorientation maneuver about an eigenaxis along the initial quaternion vector $\boldsymbol{q}(0)$.

Euler's rotational theorem is only concerned with the kinematics of the eigenaxis rotation, and it does not deal with the dynamics of the eigenaxis rotation. However, the following eigenaxis rotation theorem will describe the complete closed-loop rotational dynamics of a rigid spacecraft.

Theorem 1: Consider the closed-loop rotational motion of a rigid spacecraft with the quaternion feedback control logic (8) described by

$$\dot{\boldsymbol{\omega}} = -k\boldsymbol{q} - c\boldsymbol{\omega} \tag{9a}$$

$$\dot{\boldsymbol{q}} = -\tfrac{1}{2}\boldsymbol{\omega} \times \boldsymbol{q} + \tfrac{1}{2}q_4\boldsymbol{\omega} \tag{9b}$$

$$\dot{q}_4 = -\tfrac{1}{2}\boldsymbol{\omega}^T\boldsymbol{q} \tag{9c}$$

The solution $\boldsymbol{\omega}(t)$ and $\boldsymbol{q}(t)$ of the closed-loop system dynamics described by Eqs. (9) are collinear with $\boldsymbol{q}(0)$ for all $t \geq 0$; that is, the resulting rotational motion is an eigenaxis rotation about $\boldsymbol{q}(0)$, if and only if $\boldsymbol{\omega}(0)$ and $\boldsymbol{q}(0)$ are collinear at $t = 0$.

Proof: A rigorous proof is omitted here, but notice that $\boldsymbol{\omega} \times \boldsymbol{q} = 0$ for all $t \geq 0$ if and only if $\boldsymbol{\omega}(t)$ and $\boldsymbol{q}(t)$ are collinear for all $t \geq 0$.

Theorem 2: If the angular velocity vector $\boldsymbol{\omega}(t)$ lies along the direction of $\boldsymbol{q}(0)$, i.e.,

$$\boldsymbol{\omega}(t) = a(t)\boldsymbol{q}(0)$$

where $a(t)$ is a scalar function with $a(0) = 0$, then $\boldsymbol{q}(t)$ of Eqs. (9) will remain along the same direction of $\boldsymbol{q}(0)$; i.e., the resulting motion is an eigenaxis rotation about $\boldsymbol{q}(0)$.

Proof: This is a special case of Theorem 1 with $\boldsymbol{\omega}(0) = 0$.

IV. Cascade-Saturation Control Logic

Consider the rotational equations of motion of a rigid spacecraft described by

$$\dot{\boldsymbol{q}} = \boldsymbol{f}(\boldsymbol{q}, \boldsymbol{\omega}) = -\tfrac{1}{2}\boldsymbol{\omega} \times \boldsymbol{q} \pm \tfrac{1}{2}\sqrt{1 - \|\boldsymbol{q}\|^2}\,\boldsymbol{\omega} \tag{10a}$$

$$\dot{\boldsymbol{\omega}} = \boldsymbol{g}(\boldsymbol{\omega}, \boldsymbol{u}) = \boldsymbol{J}^{-1}(-\boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{u}) \tag{10b}$$

where $\boldsymbol{J}$ is the inertia matrix, $\boldsymbol{q} = [q_1\ \ q_2\ \ q_3]^T$ is the quaternion vector, $\boldsymbol{\omega} = [\omega_1\ \omega_2\ \omega_3]^T$ is the angular velocity vector, $\boldsymbol{u} = [u_1\ u_2\ u_3]^T$ is the control input vector, and

$$\|\boldsymbol{q}\|^2 \equiv \boldsymbol{q}^T\boldsymbol{q} = q_1^2 + q_2^2 + q_3^2$$

The state vector of the system, denoted by $\boldsymbol{x}$, is then defined as

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \boldsymbol{\omega} \end{bmatrix}$$

A dynamical system described by a set of differential equations of the form (10) is called a cascaded system since $\boldsymbol{q}$ does not appear in Eq. (10b). A cascade-saturation controller has been studied for such a cascaded system in Refs. 5–8. Saturation functions employed for the cascade-saturation controller can be defined as follows.

Definition 1: A saturation function of an n-dimensional vector $\boldsymbol{x} = [x_1, \ldots, x_n]^T$ is defined as

$$\mathrm{sat}(\boldsymbol{x}) = \begin{bmatrix} \mathrm{sat}_1(x_1) \\ \mathrm{sat}_2(x_2) \\ \vdots \\ \mathrm{sat}_n(x_n) \end{bmatrix}$$

where

$$\underset{i}{\mathrm{sat}}(x_i) = \begin{cases} x_i^+ & \text{if } x_i > x_i^+ \\ x_i & \text{if } x_i^- \leq x_i \leq x_i^+ \\ x_i^- & \text{if } x_i < x_i^- \end{cases} \tag{11}$$

In this paper, the normalized lower and upper bounds are selected as ± 1 for all i.

Similarly, a signum function of an n-dimensional vector $\boldsymbol{x}$ is defined as

$$\operatorname{sgn}(\boldsymbol{x}) = \begin{bmatrix} \operatorname{sgn}(x_1) \\ \operatorname{sgn}(x_2) \\ \vdots \\ \operatorname{sgn}(x_n) \end{bmatrix}$$

where

$$\operatorname{sgn}(x_i) = \begin{cases} +1 & \text{if } x_i > 0 \\ 0 & \text{if } x_i = 0 \\ -1 & \text{if } x_i < 0 \end{cases} \tag{12}$$

Definition 2: A normalized saturation function of an n-dimensional vector $\boldsymbol{x}$ is defined as

$$\underset{\sigma}{\operatorname{sat}}(\boldsymbol{x}) = \begin{cases} \boldsymbol{x} & \text{if } \sigma(\boldsymbol{x}) < 1 \\ \boldsymbol{x}/\sigma(\boldsymbol{x}) & \text{if } \sigma(\boldsymbol{x}) \geq 1 \end{cases} \tag{13}$$

where $\sigma(\boldsymbol{x})$ is a positive scalar function of $\boldsymbol{x}$, which characterizes the largeness of the vector $\boldsymbol{x}$.

Since the largeness of a vector $\boldsymbol{x}$ is often characterized by its norms, we may choose $\sigma(\boldsymbol{x}) = \|\boldsymbol{x}\|_2 = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$ or $\sigma(\boldsymbol{x}) = \|\boldsymbol{x}\|_\infty = \max_i |x_i|$. Note that the normalized saturation of a vector $\boldsymbol{x}$ as just defined has the same direction of the vector $\boldsymbol{x}$ itself before saturation; i.e., it maintains the direction of the vector.

Definition 3: A state feedback controller of the following form is called the m-layer cascade-saturation controller

$$\boldsymbol{u} = \boldsymbol{Q}_m \operatorname{sat}\{\boldsymbol{P}_m\boldsymbol{x} + \cdots + \boldsymbol{Q}_2 \operatorname{sat}[\boldsymbol{P}_2\boldsymbol{x} + \boldsymbol{Q}_1 \operatorname{sat}(\boldsymbol{P}_1\boldsymbol{x})]\} \tag{14}$$

where $\boldsymbol{P}_i$ and $\boldsymbol{Q}_i$ are the controller gain matrices to be properly determined. If $\boldsymbol{Q}_i$ and $\boldsymbol{P}_i$ are diagonal matrices, then we have an m-layer decentralized cascade-saturation controller.

The control logic described by Eq. (14) is a generalized form of a saturation controller discussed in Ref. 5. A cascade-relay or poly-relay control algorithm similar to Eq. (14) was also suggested in Refs. 6–8 using the signum function instead of the saturation function. The simplest form of a two-layer cascade-saturation control logic for a rigid spacecraft can be expressed as

$$\boldsymbol{u} = \boldsymbol{Q}_2 \underset{\sigma}{\operatorname{sat}}[\boldsymbol{P}_2\boldsymbol{\omega} + \boldsymbol{Q}_1 \operatorname{sat}(\boldsymbol{P}_1\boldsymbol{q})] \tag{15}$$

Consider a typical rest-to-rest eigenaxis maneuver with slew rate constraint, which will be called a constrained rest-to-rest maneuver throughout this paper. The maneuver consists of the following three phases: 1) acceleration, 2) coast, and 3) deceleration. In the spin-up acceleration phase, the spacecraft will accelerate about the eigenaxis. In the coast phase, it rotates about the eigenaxis at a constant slew rate. In this coast phase, the control input and the body rates are kept in a quasisteady mode. The following lemma guarantees the existence of such a quasisteady coast phase.

Lemma 1: If a dynamical system described by Eq. (10) is exponentially stabilized by the two-layer saturation control logic (15) in a constrained rest-to-rest maneuver problem, and if there exists a sufficiently large time instant t^* such that for $t \leq t^*$, $|(\boldsymbol{P}_1\boldsymbol{q})_i| \geq 1 \;\; \forall i$, then there exists a time interval $[t, t^*]$ in which the angular velocity vector $\boldsymbol{\omega}$ is in a quasisteady mode.

Proof: As t approaches to t^*, the closed-loop system becomes

$$\dot{\boldsymbol{q}} = \boldsymbol{f}(\boldsymbol{q}, \boldsymbol{\omega}) \tag{16a}$$

$$\dot{\boldsymbol{\omega}} = \boldsymbol{g}\{\boldsymbol{\omega}, \boldsymbol{Q}_2 \underset{\sigma}{\operatorname{sat}}[\boldsymbol{P}_2\boldsymbol{\omega} + \boldsymbol{Q}_1 \operatorname{sgn}(\boldsymbol{P}_1\boldsymbol{q})]\} \tag{16b}$$

For $t \in (t^*, \infty)$ the closed-loop system becomes

$$\dot{\boldsymbol{q}} = \boldsymbol{f}(\boldsymbol{q}, \boldsymbol{\omega}) \tag{17a}$$

$$\dot{\boldsymbol{\omega}} = \boldsymbol{g}\{\boldsymbol{\omega}, \boldsymbol{Q}_2 \underset{\sigma}{\operatorname{sat}}[\boldsymbol{P}_2\boldsymbol{\omega} + \boldsymbol{Q}_1 \operatorname{sat}(\boldsymbol{P}_1\boldsymbol{q})]\} \tag{17b}$$

Since the closed-loop system in a constrained rest-to-rest maneuver problem is assumed to be exponentially stabilized, both Eqs. (16) and (17) should be exponentially stable. As a result, along the trajectory of Eq. (16) the slew rate $\|\boldsymbol{\omega}(t)\|$ should increase. If t^* is sufficiently large, $\|\boldsymbol{\omega}(t)\|$ will become a constant slew rate $\boldsymbol{\omega}^*$ of Eq. (16), whereas $\boldsymbol{\omega}^*$ satisfies

$$\boldsymbol{g}\{\boldsymbol{\omega}^*, \boldsymbol{Q}_2 \underset{\sigma}{\operatorname{sat}}[\boldsymbol{P}_2\boldsymbol{\omega}^* + \boldsymbol{Q}_1 \operatorname{sgn}(\boldsymbol{P}_1\boldsymbol{q})]\} = 0 \tag{18}$$

which is in fact algebraic equation for $\boldsymbol{\omega}^*$. If the settling time t_s is smaller than t^*, then during the time interval $[t_s, t^*]$, $\boldsymbol{\omega}$ is very close to $\boldsymbol{\omega}^*$; i.e., $\boldsymbol{\omega}$ is in a quasisteady mode (i.e., $\dot{\boldsymbol{\omega}} \approx 0$).

Lemma 2: If a dynamical system described by Eq. (10) is exponentially stabilized by Eq. (15) in a constrained rest-to-rest maneuver problem and if the following conditions are satisfied:

$$\boldsymbol{\omega}^T \boldsymbol{g}\{\boldsymbol{\omega}, \boldsymbol{Q}_2 \underset{\sigma}{\operatorname{sat}}[\boldsymbol{P}_2\boldsymbol{\omega} + \boldsymbol{Q}_1 \operatorname{sgn}(\boldsymbol{P}_1\boldsymbol{q})]\} \geq 0, \quad \text{if } \sigma(\boldsymbol{P}_1\boldsymbol{q}) \geq 1$$

$$\boldsymbol{\omega}^T \boldsymbol{g}\{\boldsymbol{\omega}, \boldsymbol{Q}_2 \underset{\sigma}{\operatorname{sat}}[\boldsymbol{P}_2\boldsymbol{\omega} + \boldsymbol{Q}_1\boldsymbol{P}_1\boldsymbol{q}]\} < 0, \quad \text{if } \sigma(\boldsymbol{P}_1\boldsymbol{q}) < 1$$

then the slew rate $\|\boldsymbol{\omega}(t)\|$ will never exceed its upper bound $\|\boldsymbol{\omega}^*\|$.

Proof: By defining $V = (1/2)\boldsymbol{\omega}^T\boldsymbol{\omega}$ as a Lyapunov function and considering the time derivative along the closed-loop trajectory and Lemma 1, we can proof this lemma.

V. Eigenaxis Rotation Under Slew Rate Constraint

Consider a rigid spacecraft that is required to maneuver about an inertially fixed axis as fast as possible, but not exceeding the specified maximum slew rate about that eigenaxis. In this section we will show that the following saturation control logic provides such a rest-to-rest eigenaxis rotation under slew rate constraint:

$$\boldsymbol{u} = -\boldsymbol{K} \operatorname{sat}(\boldsymbol{P}\boldsymbol{q}) - \boldsymbol{C}\boldsymbol{\omega} + \boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega} \tag{19}$$

where

$$\boldsymbol{K} = \operatorname{diag}(k_1, k_2, k_3)\, \boldsymbol{J}$$

$$\boldsymbol{P} = \operatorname{diag}(p_1, p_2, p_3)$$

$$\boldsymbol{C} = c\boldsymbol{J}$$

and k_i, p_i, and c are all positive scalar constants that are the control design parameters to be properly determined. Notice the similarity between this saturation control logic and the eigenaxis slew control logic (8).

The closed-loop attitude dynamics of a rigid spacecraft employing the saturation control logic (19) are then described by

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{K} \operatorname{sat}(\boldsymbol{P}\boldsymbol{q}) - \boldsymbol{C}\boldsymbol{\omega} \tag{20a}$$

$$\dot{\boldsymbol{q}} = -\tfrac{1}{2}\boldsymbol{\omega} \times \boldsymbol{q} + \tfrac{1}{2}q_4\boldsymbol{\omega} \tag{20b}$$

$$\dot{q}_4 = -\tfrac{1}{2}\boldsymbol{\omega}^T\boldsymbol{q} \tag{20c}$$

The following lemma and theorem characterize the rotational motion of a rigid spacecraft described by Eqs. (20).

Lemma 3: Let $\dot{\theta}_{\max}$ be the maximum slew rate about an eigenaxis allowed by saturating rate gyros (i.e., $|\dot{\theta}(t)| \leq \dot{\theta}_{\max}$), and $\boldsymbol{q}(0) \neq 0$ is given with $\boldsymbol{\omega}(0) = 0$ for a rest-to-rest maneuver. It is assumed that t^* is a time instant at which there exists at least one axis such that $|p_i q_i(t^*)| = 1$, and $q_i(0)q_i(t) > 0$ for all i and for all $t \in [0, t^*]$. If we choose

$$k_i = c\frac{|q_i(0)|}{\|\boldsymbol{q}(0)\|}\dot{\theta}_{\max} \tag{21a}$$

$$\boldsymbol{K}\boldsymbol{P} = k\boldsymbol{J} \tag{21b}$$

where $k \equiv k_i p_i$ is a positive scalar constant, then we have the following results for all $t \in [0, t^*]$:

1) The rotational motion described by Eq. (20) is an eigenaxis rotation about $\boldsymbol{q}(0)$.

2) The actual slew rate about the eigenaxis is bounded as

$$\|\boldsymbol{\omega}(t)\| \leq \dot{\theta}_{\max}$$

and it increases monotonically.

3) The attitude error $\|q(t)\|$ decreases monotonically.
4) At time t^*, we have

$$|p_i q_i(t^*)| = 1 \quad \text{for all } i = 1, 2, 3$$

Proof: 1) Substituting Eq. (21) into Eq. (20), we obtain

$$\dot{\omega} = -\frac{c\dot{\theta}_{\max}}{\|q(0)\|} q(0) - c\omega \tag{22}$$

The solution of Eq. (22) can be expressed as

$$\omega(t) = e^{-ct}\omega(0) - \int_0^t e^{-c(t-\tau)} \frac{c\dot{\theta}_{\max}}{\|q(0)\|} q(0)\, d\tau \tag{23}$$

Since we are concerned with a rest-to-rest maneuver [i.e., $\omega(0)=0$], Eq. (23) is rewritten as

$$\omega(t) = -f(t)q(0) \tag{24}$$

where

$$f(t) = \frac{(1 - e^{-ct})\dot{\theta}_{\max}}{\|q(0)\|} > 0 \tag{25}$$

Since $\omega(t)$ lies along $q(0)$, Theorem 2 implies that $q(t)$ also lies entirely along $q(0)$; i.e., there exists a scalar function $g(t)$ such that

$$q(t) = g(t)q(0) \tag{26}$$

for all $t > 0$. This means that the resulting motion is an eigenaxis rotation.

2) From Eqs. (24) and (25), we obtain

$$\|\omega(t)\| = \sqrt{\omega^T\omega} = (1 - e^{-ct})\dot{\theta}_{\max}$$

which is obviously less than $\dot{\theta}_{\max}$ and $\|\omega(t)\|$ increases monotonically.

3) Substituting Eq. (26) and Eq. (24) into the quaternion kinematic differential equations, we obtain

$$\dot{g}(t) = -\tfrac{1}{2}q_4 f(t) \tag{27}$$

provided that $q_4 > 0$. The solution of Eq. (27) can then be expressed as

$$g(t) = 1 - \frac{1}{2}\int_0^t q_4(\tau) f(\tau)\, d\tau \tag{28}$$

From Eqs. (25) and (27), we obtain

$$\dot{g}(t) < 0$$

and $\|q(t)\|$ decreases monotonically.

4) Since $k_i p_i = k$ for $i = 1, 2, 3$, we have

$$p_i = \frac{k\|q(0)\|}{c|q_i(0)|\dot{\theta}_{\max}}$$

and

$$|p_i q_i(t)| = \frac{k\|q(0)\|}{c\dot{\theta}_{\max}}|g(t)|$$

Furthermore there exists a time instant t^* satisfying

$$|g(t^*)| = \frac{c\dot{\theta}_{\max}}{k\|q(0)\|}$$

such that $|p_i q_i(t^*)| = 1$ for all i; i.e., all of the elements of $q(t)$ depart the coasting phase at the same time.

The preceding lemma characterizes the properties of the closed-loop system described by Eq. (20) for $t \in [0, t^*]$. The following theorem characterizes the closed-loop system described by Eq. (20) in the entire time interval $0 \le t < +\infty$.

Theorem 3: The closed-loop system described by Eq. (20) has the following properties:

1) The entire time interval $[0, +\infty)$ consists of the three motion phases, called the acceleration, coast, and deceleration phases, with the time intervals $[0, t_s]$, $[t_s, t^*]$ and $[t^*, \infty)$, respectively, whereas t_s and t^* can be approximated as

$$t_s \approx (4/c)$$

$$t^* \approx t_s + \frac{2}{\dot{\theta}_{\max}} \tan^{-1}\left\{\frac{\|q(0)\|}{q_4(t_s)}\right\}$$

2) The resulting rotational motion is an eigenaxis rotation for all $t \in [0, \infty)$.

3) The quaternion vector $q(t)$ and the angular velocity vector $\omega(t)$ become zero as t approaches ∞.

4) The slew rate is bounded by $\dot{\theta}_{\max}$ for a properly chosen c.

Proof: 1) From Lemma 2, we have the three motion phases: the acceleration, coast, and deceleration phases. Since the angular velocity has the form (24) and the slew rate approaches $\dot{\theta}_{\max}$ with the time constant $1/c$, the settling time for this acceleration phase can be approximated as t_s. For $t \le t_s$, $\|\omega(t)\|$ increases fast and the acceleration phase is $[0, t_s]$.

Since $1 - e^{-ct} \approx 1$ for $t \ge t_s$, ω is in a quasisteady mode for $t \ge t_s$. Hence the coast phase starts from $t = t_s$.

Assuming that the coast phase ends at $t = t^*$, we can estimate t^* as follows. During the coast phase, ω is very slowly changing, and we can approximate it as

$$\omega(t) \approx -\frac{\dot{\theta}_{\max}}{\|q(0)\|} q(0) = \omega^*$$

Substituting ω^* into the quaternion kinematic differential equations of the following form:

$$\dot{\hat{q}} = f(\hat{q}, \omega) \tag{29}$$

where $\hat{q} = [q_1\ q_2\ q_3\ q_4]^T$ and

$$f(\hat{q}, \omega) = F(\omega)\hat{q}$$

$$F(\omega) = \frac{1}{2}\begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}$$

and simplifying the resulting equations, we obtain the following equation:

$$\ddot{\hat{q}} = F(\omega^*)\dot{\hat{q}} = F(\omega^*)F(\omega^*)\hat{q} = -[\dot{\theta}_{\max}/2]^2\hat{q}$$

Since the motion is an eigenaxis rotation at $t = t_s$, we have

$$\dot{q}(t_s) = \frac{1}{2}q_4(t_s)\omega^* = -\frac{\dot{\theta}_{\max} q_4(t_s)}{2\|q(0)\|} q(0)$$

For $t_s < t \le t^*$, we have

$$q(t) = \sqrt{1 + \frac{q_4^2(t_s)}{\|q(0)\|^2}}\ \sin[\dot{\theta}_{\max}(t - t_s)/2 - \phi] q(0) \tag{30}$$

where

$$\phi = \tan^{-1}\left\{\frac{\|q(0)\|}{q_4(t_s)}\right\}$$

At the end of the coast phase, $q(t)$ becomes nearly zero, and we have

$$t^* \approx t_s + (2\phi/\dot{\theta}_{\max}) \tag{31}$$

For $t \ge t^*$, we have the deceleration phase.

2) From Lemma 3, we know that all of the components of $q(t)$ reach the linear range of the saturation function at the same time;

i.e., the closed-loop equations for $t \geq t^*$ are the same as Eq. (9). At $t = t^*$, $\boldsymbol{q}(t^*)$ and $\boldsymbol{\omega}(t^*)$ lie along the vector $\boldsymbol{q}(0)$. From Theorem 1, the rotational motion is an eigenaxis rotation. Also the first part of Lemma 3 leads to the following conclusion: the closed-loop rotational motion is in fact an eigenaxis rotation for all $t \in [0, \infty)$.

3) For the deceleration phase, consider the closed-loop system (9) and a positive definite function of the form

$$V = (1/2k)V_\omega + V_q$$

where the quaternion Lyapunov function V_q and the angular velocity Lyapunov function V_ω are defined as follows:

$$V_q = q_1^2 + q_2^2 + q_3^2 + (1 - q_4)^2 \tag{32}$$

$$V_\omega = \tfrac{1}{2}\left(\omega_1^2 + \omega_2^2 + \omega_3^2\right) \tag{33}$$

The time derivative of V along the closed-loop trajectory (9) becomes

$$\dot{V} = -(c/k)\boldsymbol{\omega}^T\boldsymbol{\omega} < 0$$

This means that the closed-loop system is globally asymptotically stable, and $\boldsymbol{q}(t)$ and $\boldsymbol{\omega}(t)$ will become zero.

However, for $t \geq t^*$, we have

$$\dot{V}_q = \boldsymbol{q}^T\boldsymbol{\omega}$$

$$\dot{V}_\omega = -c\boldsymbol{\omega}^T\boldsymbol{\omega} - k\boldsymbol{q}^T\boldsymbol{\omega}$$

If $\dot{V}_q < 0$, then it is possible to have $\dot{V}_\omega > 0$; i.e., it is possible for the rate limitation to be violated. In the following, it will be shown that this situation can be avoided by properly choosing c.

4) The closed-loop stability is guaranteed by step 3 during the deceleration phase, but the rate constraint is not necessarily guaranteed as discussed earlier. Since the maneuver is an eigenaxis rotation, the angular rate equation in Eq. (9) can be simplified as

$$\ddot{\theta} + c\dot{\theta} + k\sin(\theta/2) = 0 \quad \text{for } t \geq t^* \tag{34}$$

with the following initial condition:

$$\theta(t^*) = 2\sin^{-1}[g(t^*)]$$

where θ is the rotational angle about the eigenaxis.

At the end of the coast phase, it is reasonable to assume that θ is small. Consequently, we have

$$\ddot{\theta} + c\dot{\theta} + k(\theta/2) = 0 \quad \text{for } t \geq t^*$$

Then we can properly choose k and c as follows:

$$k = 2\omega_n^2$$

$$c = 2\zeta\omega_n$$

where ζ and ω_n are the desired damping ratio and the natural frequency that characterize the second-order dynamics of the desired slew rate during the deceleration phase. It is clear that the slew rate will not exceed $\dot{\theta}_{\max}$ for all $t \in [0, \infty)$ if we do not choose a small ζ.

VI. Slew Rate and Control Constraints

We now consider a rigid spacecraft that is required to maneuver about an inertially fixed axis as fast as possible, but within the saturation limits of rate gyros as well as reaction wheels.

Let τ_i denote the torque generated by the ith reaction wheel, and also assume that

$$|\tau_i| \leq \bar{\tau}_i, \qquad i = 1, \ldots, \ell \tag{35}$$

where ℓ is the number of the reaction wheels and $\bar{\tau}_i$ are the maximum torque of ith reaction wheel. Usually, $\ell \geq 3$ to allow the failure of at most $\ell - 3$ reaction wheels.

The control torque inputs, u_1, u_2, and u_3 along the body-fixed control axes, are generated by reaction wheels, and in general the control input vector $\boldsymbol{u}$ can be expressed as

$$\boldsymbol{u} = \boldsymbol{a}_1\tau_1 + \boldsymbol{a}_2\tau_2 + \cdots + \boldsymbol{a}_l\tau_\ell$$

where $\boldsymbol{a}_i \in \mathcal{R}^3$ is the torque distribution vector of the ith reaction wheel and $\boldsymbol{a}_i^T\boldsymbol{a}_i = 1$ for all i. The torque distribution matrix is defined as

$$\boldsymbol{A} = [\boldsymbol{a}_1 \; \boldsymbol{a}_2 \; \ldots \; \boldsymbol{a}_\ell]$$

and $\boldsymbol{u} = \boldsymbol{A\tau}$. At least three column vectors of $\boldsymbol{A}$ must be linearly independent or $(\boldsymbol{AA}^T)^{-1}$ must exist for independent three-axis control.

For the commanded control input vector $\boldsymbol{u}_c$, the reaction wheel torque vector is determined as

$$\boldsymbol{\tau}_c = \boldsymbol{\Phi u}_c$$

where $\boldsymbol{\Phi} = \boldsymbol{A}^T(\boldsymbol{AA}^T)^{-1}$ is the pseudo-inverse transformation matrix. For an ideal case without actuator dynamics and saturation , the reaction wheel torque vector $\boldsymbol{\tau} = \boldsymbol{\tau}_c$ is then physically redistributed as $\boldsymbol{u} = \boldsymbol{A\tau}$, and the spacecraft will be acted by the control input vector $\boldsymbol{u}$ that is the same as the commanded control input vector $\boldsymbol{u}_c$. However, if a torque saturation occurs in one of reaction wheels, then $\boldsymbol{u} \neq \boldsymbol{u}_c$ and $\boldsymbol{\tau} \neq \boldsymbol{\tau}_c$.

Now we consider a control logic that accommodates possible torque saturation of reaction wheels but that still provides an eigenaxis rotation under slew rate constraint.

The commanded control input vector $\boldsymbol{u}_c$ that accommodates the slew rate constraint is given as

$$\boldsymbol{u}_c = -\boldsymbol{K}\text{sat}(\boldsymbol{Pq}) - \boldsymbol{C\omega}$$

where

$$\boldsymbol{P} = \text{diag}(p_1, p_2, p_3)$$

$$\boldsymbol{K} = \text{diag}(k_1, k_2, k_3)\,\boldsymbol{J}$$

$$\boldsymbol{C} = c\boldsymbol{J}$$

and furthermore we choose

$$k_i = c\frac{|q_i(0)|}{\|\boldsymbol{q}(0)\|}\dot{\theta}_{\max} \tag{36a}$$

$$\boldsymbol{KP} = k\boldsymbol{J} \tag{36b}$$

where k is a positive scalar number.

The reaction wheel torque vector $\boldsymbol{\tau}_c$ needed for $\boldsymbol{u}_c$ is then determined as

$$\boldsymbol{\tau}_c = \boldsymbol{\Phi u}_c$$

To keep the reaction wheel torque vector $\boldsymbol{\tau}$ even in the presence of saturation lying in the same direction as $\boldsymbol{\tau}_c$, we use the maximum value of the components of $\boldsymbol{\tau}_c$ to normalize it. Thus, we choose the following criterion:

$$\sigma(\boldsymbol{q}, \boldsymbol{\omega}) = \|\boldsymbol{T\tau}_c\|_\infty = \max_i |(\boldsymbol{T\tau}_c)_i| \tag{37}$$

where

$$\boldsymbol{T} = \text{diag}(1/\bar{\tau}_1, 1/\bar{\tau}_2, \ldots, 1/\bar{\tau}_\ell)$$

and the actual reaction wheel torque vector to be commanded is determined as

$$\boldsymbol{\tau} = \underset{\sigma}{\text{sat}}(\boldsymbol{\tau}_c) = \begin{cases} \boldsymbol{\tau}_c & \text{if } \sigma(\boldsymbol{q}, \boldsymbol{\omega}) \leq 1 \\ \boldsymbol{\tau}_c/\sigma(\boldsymbol{q}, \boldsymbol{\omega}) & \text{if } \sigma(\boldsymbol{q}, \boldsymbol{\omega}) > 1 \end{cases}$$

The control input vector $\boldsymbol{u}$ acting on the spacecraft, which is generated by the saturated torque vector $\boldsymbol{\tau}$, becomes

$$\boldsymbol{u} = \boldsymbol{A\tau} = \boldsymbol{A}\underset{\sigma}{\text{sat}}(\boldsymbol{\tau}_c) = \boldsymbol{A}\underset{\sigma}{\text{sat}}(\boldsymbol{\Phi u}_c)$$

and thus we have

$$u = \operatorname*{sat}_{\sigma}(u_c)$$

Finally, we have the following expression of the saturation control logic:

$$u = -\operatorname*{sat}_{\sigma}[K\text{sat}(Pq) + C\omega] \tag{38}$$

Case of Negligible Gyroscopic Coupling

The gyroscopic term is not significant during most practical rotational maneuvers, and the term can be neglected without much impact on performance and stability. For such a case, let

$$\mu(q, \omega) = \begin{cases} 1 & \text{if } \sigma(q, \omega) \le 1 \\ \sigma(q, \omega) & \text{if } \sigma(q, \omega) > 1 \end{cases}$$

Then the closed-loop dynamics of a rigid spacecraft during the acceleration and coast phases can be described by

$$\mu(q, \omega)\dot{\omega} = -\frac{c\dot{\theta}_{\max}}{\|q(0)\|}q(0) - c\omega \tag{39a}$$

$$\dot{q} = -\tfrac{1}{2}\omega \times q + \tfrac{1}{2}q_4\omega \tag{39b}$$

$$\dot{q}_4 = -\tfrac{1}{2}\omega^T q \tag{39c}$$

From the definition of $\sigma(\cdot)$, we have

$$\mu(q, \omega) = a + b^T\omega$$

where a is a scalar constant and b is a 3×1 column vector. If $\sigma(q, \omega) \le 1$, then $a = 1$ and $b = [0\ \ 0\ \ 0]^T$; otherwise there are no limitations on a and b.

The solution $\bar{\omega}(t)$ of the homogeneous differential equation (39a) during the acceleration phase satisfies

$$(a + b^T\bar{\omega})\dot{\bar{\omega}} = -c\bar{\omega}$$

and it could be written as

$$\bar{\omega}(t) = \bar{f}(t)\bar{\omega}(0)$$

Hence the solution of Eq. (39) can be expressed as

$$\omega(t) = \bar{f}(t)\omega(0) - \int_0^t \bar{f}(t - \tau)\frac{c\dot{\theta}_{\max}}{\|q(0)\|}q(0)\,\mathrm{d}\tau$$

If $\omega(0) = 0$, we obtain

$$\omega(t) = \hat{f}(t)q(0)$$

From Theorem 2, we have

$$q(t) = \hat{g}(t)q(0)$$

i.e., the maneuver during the acceleration and coast phases is still an eigenaxis rotation even in the presence of control torque saturation.

During the deceleration phase, the closed-loop system is described by

$$\mu(q, \omega)\dot{\omega} = -kq - c\omega \tag{40a}$$

$$\dot{q} = -\tfrac{1}{2}\omega \times q + \tfrac{1}{2}q_4\omega \tag{40b}$$

$$\dot{q}_4 = -\tfrac{1}{2}\omega^T q \tag{40c}$$

Similar to Theorem 1, the resulting motion is still an eigenaxis rotation, and we have

$$\ddot{\theta} + (c/\mu)\dot{\theta} + (k/\mu)\sin(\theta/2) = 0$$

This equation is the same as Eq. (34) except the positive factor μ. Hence we have a similar result to Theorem 3; i.e., the maneuver in the deceleration phase is an eigenaxis rotation, q and ω will be regulated to zero, and the magnitude of the angular velocity will never exceed $\dot{\theta}_{\max}$ for a properly chosen c.

Case of Significant Gyroscopic Coupling

The closed-loop system including the gyroscopic coupling term is described by

$$J\dot{\omega} = -\omega \times J\omega - \operatorname*{sat}_{\sigma}[K\text{sat}(Pq) + C\omega] \tag{41a}$$

$$\dot{q} = -\tfrac{1}{2}\omega \times q + \tfrac{1}{2}q_4\omega \tag{41b}$$

$$\dot{q}_4 = -\tfrac{1}{2}\omega^T q \tag{41c}$$

During the acceleration and coast phases, we have

$$\mu(q, \omega)\dot{\omega} = -\mu(q, \omega)J^{-1}\omega \times J\omega - \frac{c\dot{\theta}_{\max}}{\|q(0)\|}q(0) - c\omega$$

and

$$|\text{sat}(Pq)_i| = 1 \tag{42}$$

$$\left|\mu(q, \omega)(J^{-1}\omega \times J\omega)_i\right| < \left|\frac{c\dot{\theta}_{\max}}{\|q(0)\|}q(0)\right| \tag{43}$$

where $(\cdot)_i$ denotes the ith component of a vector $(\cdot)$. This implies that

$$\text{sgn}(\dot{\omega}_i) = -\text{sgn}(q_i), \qquad \forall i = 1, 2, 3$$

The time derivatives of V_q and V_ω along the closed-loop trajectory (41) satisfy

$$\dot{V}_q < 0 \quad \text{and} \quad \dot{V}_\omega > 0$$

and during the acceleration phase, the slew rate increases and the quaternion decreases.

During the coast phase, the angular velocity vector becomes ω^* satisfying

$$\omega^* \times J\omega^* + \operatorname*{sat}_{\sigma}[K\text{sgn}(Pq) + C\omega^*] = 0 \tag{44}$$

These are algebraic equations independent of q. During this period, the control u is near constant and we have

$$u^* = -\omega^* \times J\omega^*$$

VII. Example

Consider the near-minimum-time eigenaxis reorientation problem of the XTE spacecraft under slew rate and control torque constraints. This example will demonstrate the effectiveness and simplicity of the proposed cascade-saturation control logic.

The XTE spacecraft is equipped with four skewed reaction wheels, and the transformation matrix from the actual reaction wheel torque vector τ to the control input vector u is given as

$$A = \begin{bmatrix} \cos\alpha & -\cos\alpha & \cos\alpha & -\cos\alpha \\ \sin\alpha & 0 & -\sin\alpha & 0 \\ 0 & -\sin\alpha & 0 & \sin\alpha \end{bmatrix}$$

where the skewed angle α is 45 deg.

The maximum torque level of each reaction wheel is given as $\bar{\tau}_i = 0.3\ \text{N}\cdot\text{m}$, and the maximum slew rate is given as $\dot{\theta}_{\max} = 0.2$ deg/s, which is about 90% of the low-rate gyro measurement capability.

The inertia matrix of the XTE spacecraft is given as

$$J = \begin{bmatrix} 6292 & 0 & 0 \\ 0 & 5477 & 0 \\ 0 & 0 & 2687 \end{bmatrix} \text{kg}\cdot\text{m}^2$$

The initial quaternions for a specific reorientation maneuver are given as

$$\hat{q}(0) = [0.2652, 0.2652, -0.6930, 0.6157]^T$$

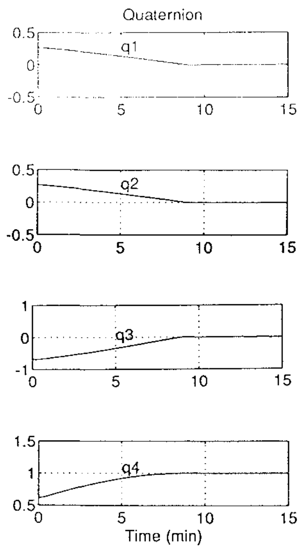

Fig. 1 Time histories of quaternions.

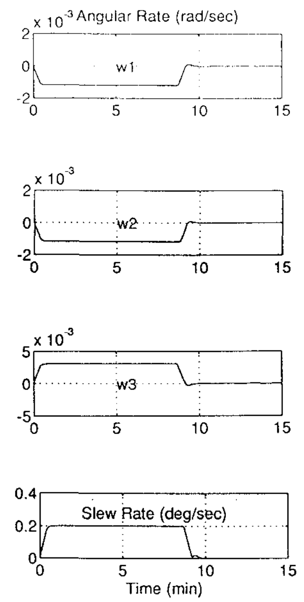

Fig. 2 Time histories of angular rates.

which corresponds to a 104-deg slew angle about the eigenaxis $q(0) = [0.2652, 0.2652, -0.6930]^T$. The slew should ideally be completed in 8.7 min.

For this case, the controller gain matrices $\boldsymbol{K}$ and $\boldsymbol{C}$ can be obtained as

$$\boldsymbol{C} = c\boldsymbol{J} = 2\zeta\omega_n\boldsymbol{J}$$

$$\boldsymbol{KP} = k\boldsymbol{J} = 2\omega_n^2\boldsymbol{J}$$

Selecting $\zeta = 0.707$ and $\omega_n = 0.1$ rad/s, we obtain

$$\boldsymbol{C} = \text{diag}(889, 774, 380)$$

$$\boldsymbol{KP} = \text{diag}(126, 110, 54)$$

From Theorem 2 and Eq. (21), we select

$$k_i = c\frac{|q_i(0)|}{\|\boldsymbol{q}(0)\|}\dot{\theta}_{\max}, \qquad i = 1, 2, 3$$

or

$$\boldsymbol{K} = \text{diag}(k_1, k_2, k_3)\boldsymbol{J} = \text{diag}(1.0452, 0.9098, 1.1667)$$

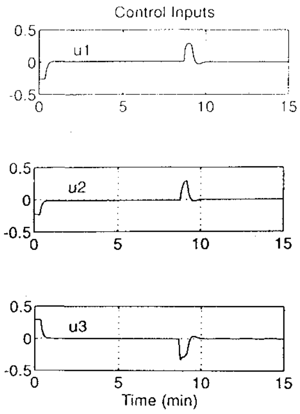

Fig. 3 Time histories of control inputs.

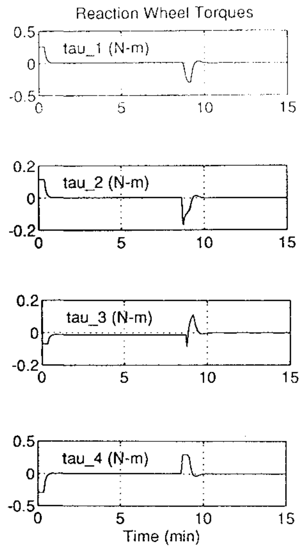

Fig. 4 Time histories of reaction wheel control torques.

The matrix $\boldsymbol{P}$ can then be determined as

$$\boldsymbol{P} = k\boldsymbol{K}^{-1}\boldsymbol{J} = \text{diag}(120, 120, 46)$$

The time histories of the closed-loop system are shown in Figs. 1–4. It can be seen that a near bang-off-bang, eigenaxis maneuver has been achieved under the slew rate and control torque constraints.

VIII. Conclusions

The problem of reorienting a rigid spacecraft as fast as possible within the physical limits of actuators and sensors has been investigated. In particular, a feedback control logic that accommodates the actuator and sensor saturation limits has been presented. The effectiveness and simplicity of the proposed nonlinear feedback control logic was demonstrated using the near-minimum-time eigenaxis reorientation problem of the XTE spacecraft under slew rate and control torque constraints.

Acknowledgments

This research was supported by NASA Goddard Space Flight Center (GSFC) through the Center for Computer-Aided Design of the University of Iowa. The authors would like to thank Frank Bauer, Tobin Anthony, and Harry Frisch of the NASA GSFC for their support and interest in this research.

References

[1]Bauer, F. H., Femiano, M. D., and Mosier, G. E., "Attitude Control System Conceptual Design for the X-Ray Timing Explorer," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, AIAA, Washington, DC, 1992, pp. 236–249.

[2]Bilimoria, K. D., and Wie, B., "Time-Optimal Three-Axis Reorientation of a Rigid Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 3, 1993, pp. 446–452.

[3]Wie, B., and Barba, P., "Quaternion Feedback for Spacecraft Large Angle Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 3, 1985, pp. 360–365.

[4]Wie, B., Weiss, H., and Arapostathis, A., "Quaternion Feedback Regulator for Spacecraft Eigenaxis Rotations," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 3, 1989, pp. 375–380.

[5]Teel, A. R., "Global Stabilization and Restricted Tracking for Multiple Integrators with Bounded Controls," *System & Control Letters*, Vol. 18, 1992, pp. 165–171.

[6]Moroz, A. I., "Time-Optimal Feedback Control for Linear Non-Stationary Systems," *International Journal of Control*, Vol. 39, No. 5, 1984, pp. 929–938.

[7]Moroz, A. I., "Constructive Time-Suboptimal Control Laws in Non-Stationary Non-Oscillatory Systems," *International Journal of Control*, Vol. 39, No. 5, 1984, pp. 939–953.

[8]Moroz, A. I., "RELION: Time-Suboptimal Variable Structure Control Law," *International Journal of Control*, Vol. 52, No. 3, 1990, pp. 655–668.